

Critical behavior of cross sections at LHC

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Abstract

Recent experimental data on elastic scattering of high energy protons show that the critical regime has been reached at LHC energies. The approach to criticality is demonstrated by increase of the ratio of elastic to total cross sections from ISR to LHC energies. At LHC it reaches the value which can result in principal change of the character of proton interactions. The treatment of new physics of hollowed toroid-like hadrons requires usage of another branch of the unitarity condition. Its further fate is speculated and interpreted with the help of the unitarity condition in combination with present experimental data. The gedanken experiments to distinguish between different possibilities are proposed.

1 Introduction

Recent experimental data on elastic scattering of high energy protons [1, 2] show quite surprising phenomenon of increase of the ratio of elastic to total cross sections with energy increase in the interval from ISR to LHC energies. This share used to decrease at lower energies but reversed the tendency at ISR (for the comparison see the tables in [3, 4]). Moreover, at LHC energies it approaches the critical value [5, 6]. For the first time, that phenomenon can reveal the transition from the branch of the unitarity condition dominated by inelastic processes where elastic scattering is treated as the shadow of inelastic collisions to the dominance of elastic scattering which would require new interpretation. Elastic scattering of polarised protons or charge asymmetries of pions produced in inelastic collisions could help in studies of different possibilities.

The information about elastic scattering comes from the measurement of the differential cross section $d\sigma/dt$ at some energy s as a function of the

transferred momentum t at its experimentally available values. It is related to the scattering amplitude $f(s, t)$ in a following way

$$\frac{d\sigma}{dt} = |f(s, t)|^2. \quad (1)$$

The variables s and t are the squared energy E and transferred momentum of colliding protons in the center of mass system $s = 4E^2 = 4(p^2 + m^2)$, $-t = 2p^2(1 - \cos \theta)$ at the scattering angle θ . From this measurement one gets the knowledge only about the modulus of the amplitude. The interference between the nuclear and Coulomb contributions to the amplitude f allows to find out the ratio of the real and imaginary parts of the elastic scattering amplitude $\rho(s, t) = \text{Re}f(s, t)/\text{Im}f(s, t)$ just in forward direction $t = 0$ $\rho(s, 0) = \rho_0$ but not at any other values of t .

The typical shape of the differential cross section at high energies contains the exponentially decreasing (with increase of $|t|$) diffraction cone with energy dependent slope $B(s)$ and more slowly decreasing tail at larger transferred momenta with much smaller values of the cross section.

2 The unitarity condition

The most stringent and reliable information about the amplitude f comes from the unitarity of the S -matrix

$$SS^+ = 1 \quad (2)$$

or for the scattering matrix T ($S = 1 + iT$)

$$2\text{Im}T_{ab} = \Sigma_n \int T_{an}T_{nb}^* d\Phi_n, \quad (3)$$

where the whole n -particle phase space Φ_n is integrated over. It relates the amplitude of elastic scattering $f = T_{22}$ to the amplitudes of n -particle inelastic processes T_{2n} declaring that the total probability of all outcomes of the interaction must be equal 1¹. In the s -channel this indubitable condition is usually expressed in the form of the well known integral relation (for more details see, e.g., [7, 8, 3]). This relation can be simplified to the algebraic one

¹The non-linear contribution from the elastic amplitude appears in the right-hand side for $n = 2$.

using the Fourier – Bessel transform of the amplitude f which retranslates the momentum data to the shortest transverse distance between the trajectories of the centers of colliding protons called impact parameter b and is written as

$$i\Gamma(s, b) = \frac{1}{2\sqrt{\pi}} \int_0^\infty d|t| f(s, t) J_0(b\sqrt{|t|}). \quad (4)$$

The unitarity condition in the b -representation reads (for the review see, e.g., Refs [3, 4])

$$G(s, b) = 2\text{Re}\Gamma(s, b) - |\Gamma(s, b)|^2. \quad (5)$$

The left-hand side describes the transverse impact-parameter profile of inelastic collisions of protons (for more detailed discussion see [4, 6]). It satisfies the inequalities $0 \leq G(s, b) \leq 1$ and determines how absorptive is the interaction region depending on the impact parameter (with $G = 1$ for the full absorption and $G = 0$ for the complete transparency). The profile of elastic processes is determined by the subtrahend in Eq. (5). If $G(s, b)$ is integrated over the impact parameter, it leads to the cross section of inelastic processes. The terms on the right-hand side would produce the total cross section and the elastic cross section, correspondingly.

3 Central collisions

At the beginning, let us study the energy dependence of interaction profiles for central collisions of impinging protons at $b = 0$. Then the condition (5) is written as

$$G(s, b = 0) = \zeta(2 - \zeta), \quad (6)$$

where

$$\zeta(s) = \frac{\sigma_{tot}(s)}{4\pi B(s)} = \frac{4\sigma_{el}(s)}{(1 + \rho_0^2(s))\sigma_{tot}(s)} \approx \frac{4\sigma_{el}(s)}{\sigma_{tot}(s)} \approx (4\pi)^{-0.5} \int_0^\infty d|t| \sqrt{d\sigma/dt}. \quad (7)$$

is proportional to the experimentally measurable dimensionless ratio of the elastic cross section σ_{el} (or the diffraction cone slope B) to the total cross section σ_{tot} . It is computed integrating the experimentally measured $d\sigma/dt$ so that any approximation can easily be estimated. The approximation sign refers to the neglected factor $1 + \rho_0^2$. According to experimental data $\rho_0(7 \text{ TeV}, 0) \approx 0.145$. The parameter ζ is uniquely determined by the normalization of the amplitude f .

Thus, according to the unitarity condition the absorption for central collisions is governed by a single experimentally measured parameter ζ related to the share of elastic processes. For central ($b = 0$) collisions, the inelastic profile $G(s, 0)$ achieves the maximum value equal to 1 (the full absorption) at $\zeta(s) = 1$. It decreases parabolically $G(s, 0) = 1 - \epsilon^2$ for any decline of ζ ($\zeta = 1 \pm \epsilon$) from 1, i.e., it is very small for small decline ϵ . The positivity of $G(s, 0)$ imposes the limit $\zeta \leq 2$. At $\zeta = 2$ the complete transparency of central collisions $G(s, 0) = 0$ is achieved.

The elastic profile also reaches 1 at $b = 0$ for $\zeta = 1$ and completely saturates the total profile for $\zeta = 2$.

The experimentally measured share of elastic processes demonstrates non-trivial dependence on energy (see the Table in [3]). At low energies up to ISR the parameter ζ decreases (from about 1 down to values about 2/3) but then starts increasing and reaches its critical value 1 for 7 TeV data at LHC. It is intriguing whether this increase will really show up in experiments at higher energies or it will be saturated asymptotically with ζ tending to 1 from below. The saturation would lead to the conservative stable situation of slow approach to full absorption in central collisions while further increase will require the transition to another branch of the unitarity equation and new physics interpretation.

To explain these statements let us rewrite Eq. (6) as

$$\zeta(s) = 1 \pm \sqrt{1 - G(s, 0)}. \quad (8)$$

One used to treat elastic scattering as a shadow of inelastic processes. This statement is valid when the branch with negative sign is considered because it leads to proportionality of elastic and inelastic contributions for small $G(s, 0) \ll 1$. That is typical for electrodynamics (e.g., for processes like $ee \rightarrow ee\gamma$) and for optics. Therefore the increase of the elastic share at diminishing role of inelastic production came as a surprise. However, for strong interactions, this share is close to 1 (see the Table). The approach of ζ to 1 at 7 TeV corresponds to complete absorption in central collisions. This value is considered as a critical one because from (8) one gets significant conclusion that the excess of ζ over 1 implies that the unitary branch with positive sign is at work. This branch was first considered in [9] with application to high energy particle scattering. That changes the interpretation of the role of elastic processes as being a simple "shadow" of inelastic ones.

Some slight trend of ζ to increase and become larger than 1 can be noticed

from comparison of TOTEM data at 7 TeV [1] where it can be estimated² in the limits 1.00 and 1.02 and at 8 TeV [2] where it is about 1.04 though within the accuracy of experiments about ± 0.024 . The precise data at 13 TeV are needed. The further increase of the share of elastic scattering with energy is favored by extensive fits of available experimental information for the wide energy range and their extrapolations to ever higher energies done in Refs [10, 11] as well as by some theoretical speculations (e.g., see Ref. [12]). The asymptotical values of ζ are about 1.5 in Refs [10, 11] and 1.8 [12]. They correspond to incomplete but noticeable transparency at the center of the interaction region.

4 The shape of the inelastic interaction region

The detailed shape of the inelastic interaction region can be obtained with the help of relations (4), (5) if the behavior of the amplitude $f(s, t)$ is known. Its modulus and the ρ_0 values are obtained from experiment. The most prominent feature of $d\sigma/dt$ is its rapid exponential decrease with increasing transferred momentum $|t|$, especially in the near forward diffraction cone. Inserting the exponential shape in Eqs (7), (4) one can write

$$i\Gamma(s, b) \approx \frac{\sigma_t}{8\pi} \int_0^\infty d|t| \exp(-B|t|/2)(i + \rho) J_0(b\sqrt{|t|}). \quad (9)$$

Let us stress that the diffraction cone dominates the contribution to $\text{Re}\Gamma$ in Eqs (gam2), (ze) so strongly that the tail of the differential cross section at larger $|t|$ can be completely neglected at the level less than 0.1% by itself and it is suppressed additionally by the Bessel function J_0 . The accuracy of the approximation was estimated using fits of the experimental differential cross section outside the diffraction cone by simplest analytical expressions. Moreover, it was shown [13, 14] by computing how well the versions with direct fits of experimental data and with their exponential approximation coincide if used in the unitarity condition. Therefore the expression (9) can be treated as following directly from experiment and being very precise.

²The experimental values of the ratios of elastic to total cross section and ρ_0 have been used.

Herefrom, one calculates

$$\text{Re}\Gamma(s, b) = \zeta \exp(-\frac{b^2}{2B}). \quad (10)$$

Correspondingly, the shape of the inelastic profile for small ρ_0 is given by

$$G(s, b) = \zeta \exp(-\frac{b^2}{2B})[2 - \zeta \exp(-\frac{b^2}{2B})]. \quad (11)$$

It depends on two measured quantities - the diffraction cone width $B(s)$ and its ratio to the total cross section ζ , and scales as a function of $b/\sqrt{2B}$. It has the maximum at

$$b_m^2 = 2B \ln \zeta \quad (12)$$

with maximum absorption $G(b_m) = 1$ for $\zeta \geq 1$. For $\zeta < 1$ (which is the case, e.g., at ISR energies) one gets incomplete absorption $G(s, b) < 1$ at any physical $b \geq 0$ with the largest value reached at $b = 0$ because the real maximum of G would appear at non-physical values of b for lower energies. Then the disk is semi-transparent.

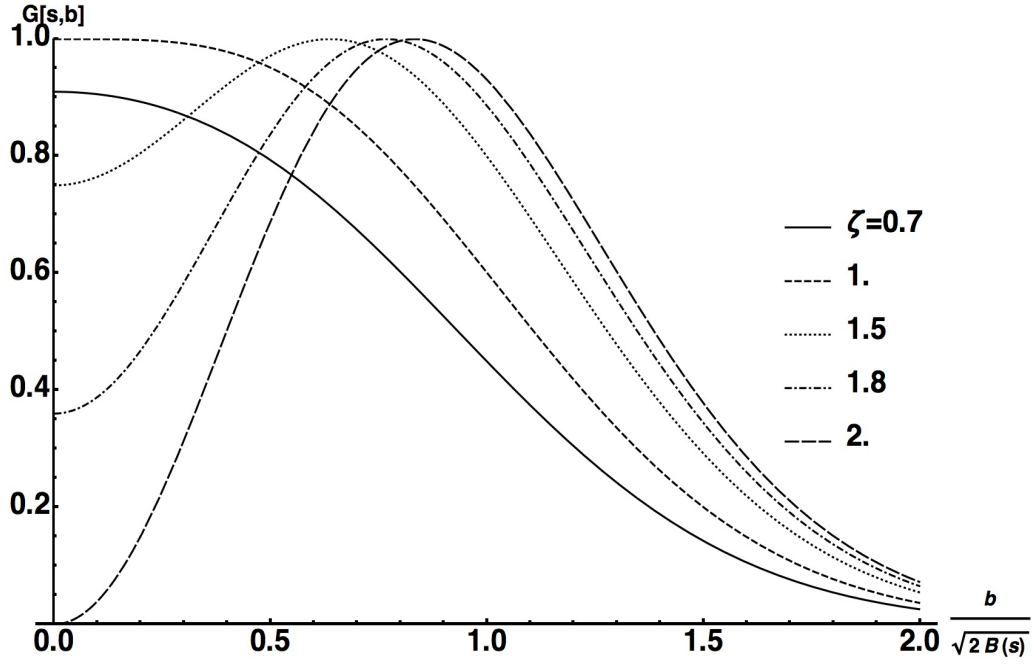
At $\zeta = 1$, which is reached at 7 TeV, the maximum is positioned exactly at $b = 0$, and maximum absorption occurs there, i.e. $G(s, 0) = 1$. The disk center becomes black. The strongly absorptive core of the inelastic interaction region grows in size compared to ISR energies (see [13]) as we see from expansion of Eq. (11) at small impact parameters:

$$G(s, b) = \zeta[2 - \zeta - \frac{b^2}{B}(1 - \zeta) - \frac{b^4}{4B^2}(2\zeta - 1)]. \quad (13)$$

The negative term proportional to b^2 vanishes at $\zeta = 1$, and $G(b)$ develops a plateau which extends to quite large values of b (about 0.5 fm). The plateau is very flat because the last term starts to play a role at 7 TeV (where $B \approx 20 \text{ GeV}^{-2}$) only for larger values of b .

With further increase of elastic scattering, i.e., at $\zeta > 1$, the maximum shifts to positive physical impact parameters. A dip is formed at $b=0$ leading to a concave shaped inelastic interaction region - approaching a toroid-like shape (see [5, 4, 15]). This dip becomes deeper at larger ζ . The limiting value $\zeta = 2$ leads to complete transparency at the center $b = 0$ as discussed in the previous section. It can be only reached if the positive sign branch of the unitarity condition is applicable.

Figure 1: The evolution of the inelastic interaction region in terms of the survival probability. The values $\zeta = 0.7$ and 1.0 correspond to ISR and LHC energies and agree well with the result of detailed fitting to the elastic scattering data [16, 13, 17]. A further increase of ζ leads to the toroid-like shape with a dip at $b = 0$. The values $\zeta = 1.5$ are proposed in [10, 11] and $\zeta = 1.8$ in [12] as corresponding to asymptotical regimes. The value $\zeta = 2$ corresponds to the "black disk" regime ($\sigma_{el} = \sigma_{in} = 0.5\sigma_{tot}$). For more discussion of the black disk and the geometrical scaling see Refs [18, 19, 20].



All these features are demonstrated in Fig. 1 borrowed from Ref. [6]. The asymptotical regimes with further increase of the share of elastic scattering proposed in Refs [10, 11, 12] predict the diminished absorption for central collisions. The whole space structure reminds the toroid (tube) with absorbing black edges which looks as if being more and more transparent for the elastic component at the very center. However, the realistic estimates of its effects at the energies 13 TeV and 100 TeV [21] show that extremely high accuracy of experiments will be necessary to observe these effects.

That is especially true because the cross sections of processes with small impact parameters are very small. Integrating the total and elastic terms in Eq. (11) up to impact parameters $b \leq r$ one estimates their roles for different radii r .

$$\sigma_{el}(s, b \leq r) = \sigma_{el}(s)[1 - \exp(-r^2/B(s))], \quad (14)$$

$$\sigma_{tot}(s, b \leq r) = \sigma_{tot}(s)[1 - \exp(-r^2/2B(s))]. \quad (15)$$

One gets that the contribution of processes at small impact parameters $b^2 \ll 2B$ diminishes quadratically at small $r \rightarrow 0$. In particular, inelastic processes contribute at $r \rightarrow 0$ as

$$\sigma_{in}(s, b \leq r) \rightarrow \pi r^2 G(s, 0) + O(r^4); \quad (r^2 \ll B). \quad (16)$$

The maximum intensity of central collisions is at $\zeta = 1$. That has been used in Ref. [14] for explanation of jets excess in very high multiplicity events at 7 TeV as an indication on the active role of gluons at that energy. It tends to 0 for $\zeta \rightarrow 2$. Thus, one predicts the diminished role of jet production from central collisions with increase of ζ . It would ask for extremely precise data to reveal any evolution of that effect at higher energies because according to estimates of Refs [10, 11] the decline from criticality is very small up to 100 TeV: $\zeta(13TeV) = 1.05 - 1.06$; $\zeta(95TeV) = 1.12 - 1.15$.

The peripheral regions dominate, especially in inelastic processes.

The spatial region of elastic scattering as derived from the subtrahend in Eq. (11) is strongly peaked in the forward direction. The contribution to the elastic cross section is suppressed at small b and comes mainly from impact parameters $b^2 \approx 2B$. The average value of the squared impact parameter for elastic scattering can be estimated as

$$\langle b_{el}^2 \rangle = \sigma_{el}(s)/\pi\zeta^2(s). \quad (17)$$

Inelastic processes are much more peripheral. The ratio of the corresponding values of squared impact parameters is

$$\frac{\langle b_{in}^2 \rangle}{\langle b_{el}^2 \rangle} = \zeta \frac{8 - \zeta}{4 - \zeta}. \quad (18)$$

This ratio exceeds 2 already at LHC energies and would become equal to 6 for (would be!) $\zeta = 2$. The peripherality of inelastic processes compared to elastic ones increases with increase of the share of elastic collisions.

5 Discussion and conclusions

The intriguing increase of the share of elastic processes to the total outcome observed at energies from ISR to LHC attracts much attention nowadays. Its approach to 1/4 at LHC can become a critical sign of the changing character of processes of proton interactions if the above tendency of increase persists. The concave central part of the inelastic interaction region would be formed. The inelastic interaction region looks like a toroid hollowed inside and strongly absorbing in its main body at the edges. The role of elastic scattering in central collision becomes increasing. That is surprising and contradicts somewhat to our theoretical prejudices. From the formal theoretical point of view it requires to consider another branch of the unitarity condition that asks for its physics interpretation.

It is hard to believe that protons become more penetrable at higher energies after being so dark in central collisions with $G(s, 0) = 1$ at 7 TeV unless some special coherence within the internal region develops. Moreover, it seems somewhat mystifying why the coherence is more significant just for central collisions but not at other impact parameters where inelastic collisions become dominant. The role of string junction in fermionic hadrons can become crucial. The relative strengths of the longitudinal and transverse components of gluon (string) fields can probably explain the new physics of hollowed hadrons.

One could imagine another classical effect that "black" protons start scattering in the opposite direction [6] like the billiard balls for head-on collisions. Snell's law admits such situation for equal reflective indices of colliding bodies. That can be checked if forward and backward scattered protons can be distinguished in experiment. Then they should wear different labels. One

could use the proton spin as such a label. In principle, experiments with polarised protons can resolve the problem. However, the more realistic classical scenario in this case would be complete breaking the balls into pieces, i.e. dominance of inelastic processes.

Another hypothesis [22] treats the hollowed internal region as resulting from formation of cooler disoriented chiral condensate inside it ("baked-alaska" DCC). The signature of this squeezed coherent state would be some disbalance between the production of charged and neutral pions [23] noticed in some cosmic ray experiments. However the cross sections for central collisions seem to be extremely small as discussed above. The failure to find such events at Fermilab is probably connected with too low energies available. It leaves some hope for higher energies in view of discussions above. Total internal reflection of coherent states from dark edges of the toroid can be blamed for enlarged elastic scattering (like transmission of laser beams in optical fibers).

The transition to the deconfined state of quarks and gluons in the central collisions could also be claimed responsible for new effects (see Ref. [24]). The optical analogy with the scattering of light on metallic surface as induced by the presence of free electrons is used. Again, it is hard to explain why that happens for central collisions while peripheral ones with impact parameters near b_m are strongly inelastic.

The last, but not the least, is the hypothesis that centrally colliding protons at $\zeta = 2$ remind solitons which "pass through one another without losing their identity. Here we have a nonlinear physical process in which interacting localized pulses do not scatter irreversibly" [25]. Non-linearity and dispersive properties (the chromopermittivity [26]) of a medium compete to produce such effect.

To conclude, the problem of increasing elastic cross section can be only solved by experiment at higher energies. If this tendency persists, one should invent new ways of explaining the transition to quite uncommon regime of proton interactions with peculiar shapes of the interaction region.

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